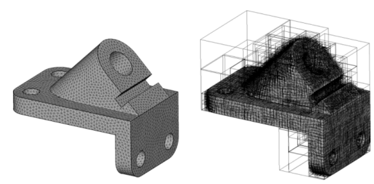
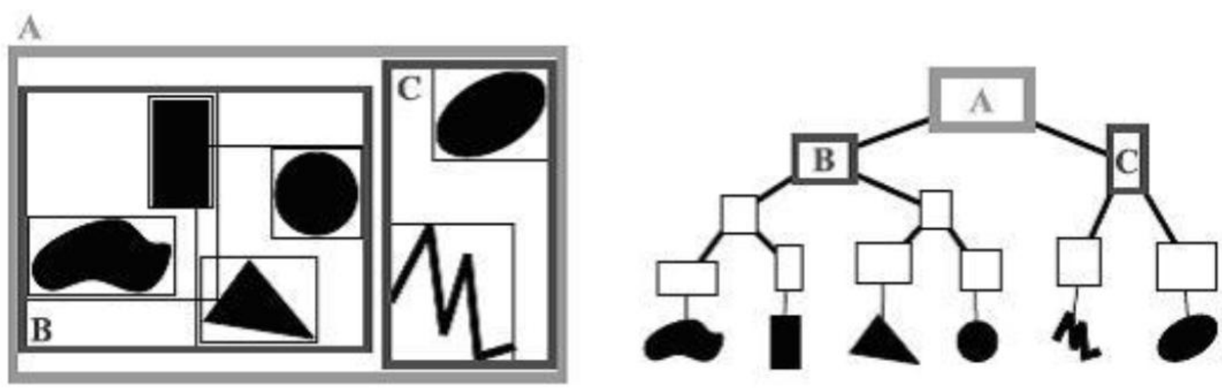
# Bounding Volume Hierarchies



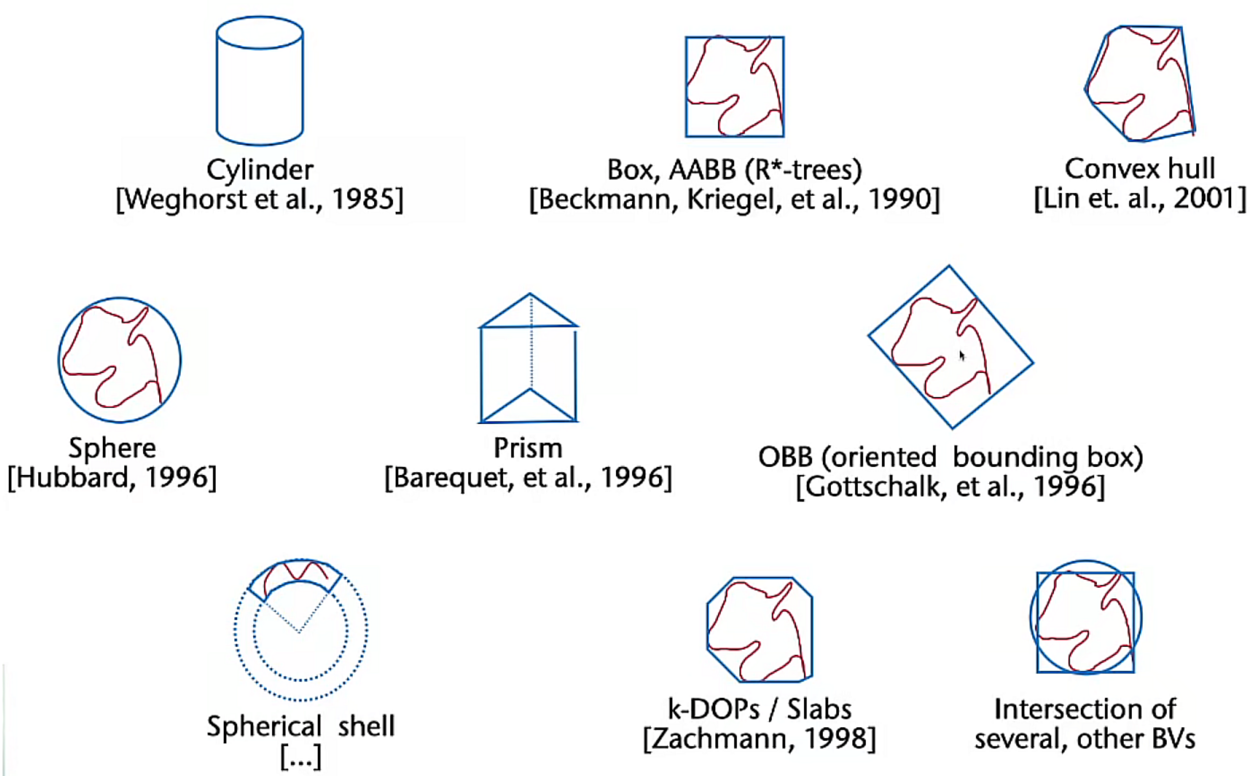
given a set $S$ of objects like polygons, if $|S| \leq k$ , $BVH(S)$ is a leaf node $b$ that stores $S$, otherwise it is a node $v$ with a number $n_v$ of children $v_1 \ldots, v_n$, where $v_i$ is the $BVH(S_i)$, where $S_i \subseteq S : \bigcup S_i = S$, i.e. is a subset of $S$ and $v$ stores a bounding volume $b_v(v)$ from a set of possible bounding volumes such that $\forall p \in S :$ $p \subseteq b_v(v)$, i.e. each polygon is completely included into the space




### types of BVH

- **layered BVH:** $\forall$ children $v_i$, $b_v(v_i) \subseteq b_v(v)$
- **wrapped BVH:** $\forall$ leaves $v_i$ of the node $v$, $S(v_i) \subseteq b_v(v)$

# types of bounding volumes

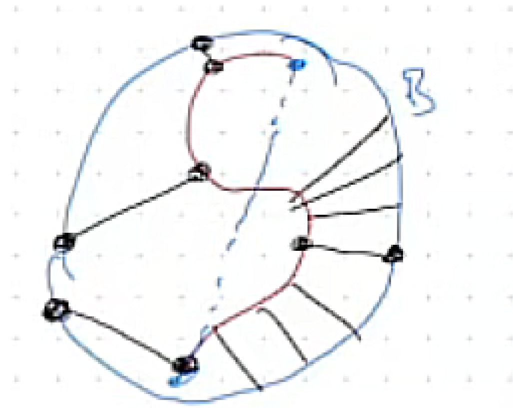



# Tightness

let $S$ be a surface or a mesh, $B$ a BV such that $S \subseteq B$, we can define the **directed Hausdoff distance** $h(B, S) = \max_{p \in B} \min_{q \in S} d(p, q)$, in general $d$ is the **L2 norm.** i.e. it is the distance between a point in $S$ and the closest border of $B$.

given the **diameter** $\mathrm{diam}(S) = \max_{p,q \in S} d(p, q)$

the **tightness** is $\tau(B, S) = \frac{h(B,S)}{\mathrm{diam}(S)}$, i.e. HD distance normalized with respect to the maximum distance between 2 points in $S$, this is in practice the relative length of the line connecting a point of $S$ to the border of $B$, where $0 \leq \tau \leq \frac{1}{2}$, where $\tau = 0$ if the point is on the border of $B$ and $\tau = \frac{1}{2}$ if the point is in the center of $B$

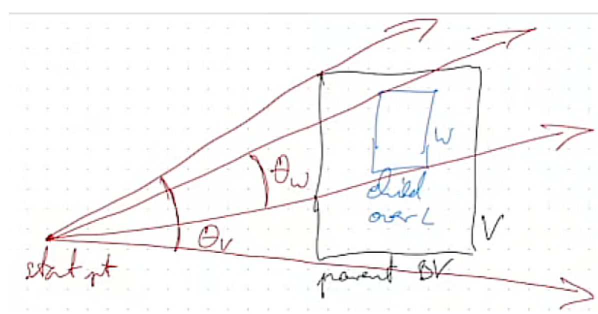



# BVH construction

given a set of polygons in 3D $S$, we consider the **midpoints** $p_i \in S$**,** we calculate the **PCA** between them and transform the $p_i$, then we compute the **median** along the most spreaded, being the **principal component**, we now partition the set of points along this and obtain 2 subsets $L, R, : S = L \cup R$

we can now improve with a **sweep plan** approach, to do this we construct a cost function $C(L,R) = P(traverse_L)C(L) + P(traverse_R)C(R)$. since those probabilities are **case-dependent** we will now consider the case of ray tracing.

let's take a bounding volume parent box, we are going to sweep a plane in the principal axis  and splitting the set to obtain a inner bounding volume, now, considering the origin of the ray we can compute all the orientations for which we can obtain an interception wit the the parent or child Bounding Volume, we end up with 2 angle spans $\theta_w, \theta_v$, the probability of a traversal is just $\frac{\theta_w}{\theta_v} \approx \frac{area(w)}{area(v)}$




it follows that

$C(L,R) = \frac{area(bbox(L))}{area(bbox(S))}C(L) + P\frac{area(bbox(R))}{area(bbox(S))}C(R)$, since the function is recursive, we can approximate it by just using the number of polygons in the sets, obtaining

$C(L,R) = \frac{area(bbox(L))}{area(bbox(S))}|L| + P\frac{area(bbox(R))}{area(bbox(S))}|R|$

what do we want now is the $\min_{L,R} C(S)$:

sort the $p_i \in S \to p_1 \ldots, p_n$

for $j = 0 \to n$:

calculate the cost $c(\{p_1 \ldots, p_j\}, \{p_{j+1} \ldots, p_n\})$

if cost is small of the minimum found: remember $j$